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Universality classes of second-order dynamical phase transitions

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The long-time asymptotic properties of numerous dynamical lattice models are described by rich phase diagrams. A suggestive pattern is observed about the universality class of second-order phase transitions in such systems: if the transition occurs from (or into) a single absorbing state (where the order parameter is then zero) then it belongs to the same universality class as directed percolation and Reggeon field theory (RFT). This has been conjectured to be always true for one-component systems by Grassberger and also by Janssen. Grinstein, Lai, and Browne have argued for a possible generalization of this rule to multicomponent systems. I present an example of a one-component lattice model where the second-order transition about a single absorbing state is not in the RFT universality class, thus violating even the weaker conjecture of Grassberger and Janssen.

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Nonequilibrium phase transitions in dynamical lattice models are the subject of growing interest [1,2]. In contrast to equilibrium phase transitions, nonequilibrium (also known as "dynamical") phase transitions are less well understood and even their classification into universality classes, or which of their properties place them into different classes, is still a basic open question.

Schlögl's [3] first model, $X \leftrightarrow 2X$, $X \to 0$, is the prototype of one-component (X) systems undergoing a second-order dynamical phase transition. Depending on the relative rates of the processes, the system may either evolve to an empty state, where there are no X left, or to a reactive steady state with a finite concentration of X (which serves as an order parameter). By "second-order," we mean that the order parameter is continuous throughout the transition. The empty phase is called an "absorbing" state, for once the system is in this state it cannot leave it.

Numerous lattice models and cellular automata are closely related to Schlögl's first model, including the contact process [4], the $A2$ model [6], the cluster transition model [7], and directed percolation (where the special dimension can be regarded as time), to mention just a few. All of these systems exhibit a second-order dynamical phase transition from a single absorbing state into a reactive steady state. The transition can be characterized by critical exponents, analogous to equilibrium phase transitions. A remarkable similarity in the numerical value of these exponents for all systems studied suggests that they belong to the same universality class.

Cardy and Sugar [8] proved that the directed percolation problem is in the same universality class as Reggeon field theory [9] (RFT). Transitions in the RFT universality class are represented by the equation

$$\frac{\partial\rho(\mathbf{x},t)}{\partial t}=D\nabla^2\rho(\mathbf{x},t)-r\rho(\mathbf{x},t)-u\rho(\mathbf{x},t)^2+\eta(\mathbf{x},t)\ , \qquad (1)$$

where $\rho(\mathbf{x},t)$ denotes the order-parameter field (which can be usually understood as a concentration of the particles) as a function of space and time. The diffusion term accounts for both intrinsic diffusion resulting from the reaction rules and for explicit diffusion of the particles. The last term η represents the noise in the system, which vanishes as the absorbing state is approached. Since the order parameter vanishes then too, the noise is commonly assumed to have the property

$$\langle \eta(\mathbf{x},t)\eta(\mathbf{x}',t')\rangle=\Gamma_\rho(\mathbf{x},t)\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')\ . \qquad (2)$$

The remaining terms on the right-hand side of Eq. (1), $-r\rho-u\rho^2$, are the only ones that survive in the mean-field approximation. They represent the birth and death of particles. As long as $r>0$ the system flows into the absorbing state $\rho=0$. The transition into the stationary, reactive phase takes place as r becomes negative.

The RFT universality class is extremely robust. In addition to models with two-particle interactions it seems to include systems with three-particle interactions, such as the $N3$ and the $D3$ models [6]. Numerical simulations of one-dimensional lattice models and cellular automata based on Schlögl's second model $2X \leftrightarrow 3X$ and $X \rightarrow 0$, for which mean-field analysis predicts a first-order phase transition, show a continuous transition with RFT critical exponents. This has led Grassberger [10] to conjecture that all one-component nonequilibrium models with a transition about a single absorbing state belong to the RFT universality class. A similar conjecture was formulated by Janssen [11].

It would be interesting to find a counterexample to the Grassberger-Janssen conjecture. Unfortunately, the critical exponents of nonequilibrium systems are usually determined numerically. The measuring of these exponents is a difficult task, often yielding inaccurate results. Indeed, the cellular automaton of Bidaux, Boccara, and Chaté [2] (BBC) was initially believed to violate the conjecture but was later found to be in the RFT class [13]. Takayasu and Tretyakov [14] have studied models of branching annihilating random walks [15] (BAW's). Their results suggest that BAW's may violate the conjecture. However, recent studies point to the contrary [16].

Consider now the following one-component process in one dimension. Each lattice site may be either empty or occupied by a particle A. The A particles can randomly hop to the nearest-neighbor site to their right or left. If the target site is already occupied, then the two particles annihilate (the original site and the target site are then empty). In addition to these diffusion and annihilation processes there is also input of A particles at a rate proportional to their global concentration on the line, ρ_A. The input is performed onto random lattice sites, Again, if the target site happens to be occupied, instantaneous annihilation takes place. Thus the system consists of diffusing A particles with a characteristic diffusion constant D, which annihilate immediately upon encounter, $A + A \rightarrow 0$. The input of particles occurs at a rate $k\rho_A$, where k is a constant.

The above system has a single absorbing state when the line is completely empty, $\rho_A = 0$. If there is no input ($k = 0$), the system will always evolve to the absorbing empty state. For any finite k, it will arrive at a steady-state behavior characterized by a finite stationary concentration ρ^s_A. I use dimensional analysis to estimate ρ^s_A. The only two physical parameters determining the long-time asymptotic behavior are the diffusion coefficient D and the input rate constant k (note that annihilation is immediate and therefore involves no characteristic time scale). Since $D \sim (\text{length})^2/(\text{time})$ and $k \sim 1/(\text{time})$, we have $\rho^s_A \sim 1/(\text{length}) \sim (k/D)^{1/2}$. Interpreting ρ^s_A as an order parameter and k as its critical field, the system is seen to violate the Grassberger-Janssen conjecture, since for it the order-parameter critical exponent is $\beta = \frac{1}{2}$, unlike the RFT value of $\beta \approx 0.277$ (in one dimension). A similar argument has been applied to diffusion-limited coagulation ($A + A \rightarrow A$) in one dimension with random particle input at constant rate R [17]. There $R \sim 1/(\text{length})(\text{time})$ and dimensional analysis yields $\rho^s_A \sim (R/D)^{1/3}$, in complete agreement with the *exactly* known result [17].

In summary, I have presented a one-component dynamical model in one dimension which violates the conjecture of Grassberger and Janssen. Recently, Grinstein, Lai, and Browne [18] have argued for a possible extension of the conjecture to multicomponent systems. Studies of the two-component Ziff-Gulari-Barshad [19] (ZGB) model show that its second-order phase transition is indeed in the RFT universality class [18,20]. The argument of Grinstein, Lai, and Browne is that all but one field can be "renormalized away" and are therefore irrelevant. The remaining relevant field satisfies equations similar to (1) and (2), placing the system in the RFT universality class. That is, Grinstein, Lai, and Browne effectively collapse multicomponent systems into a one-component system and thereafter rely on the Grassberger-Janssen conjecture. Thus the counterexample presented here also casts doubts on the stronger conjecture of Grinstein, Lai, and Browne.

In closing, I would like to draw attention to the fact that even in the cases where we seem to explain second-order transitions as RFT our understanding is far from complete. The casting of the ZGB model into a RFT equation does not fully account for the lattice's dimension. Thus, while it explains the observed RFT exponents in dimensions $d \geq 2$, the strikingly different behavior in one dimension, where the second-order phase transition does not even take place, remains a puzzle. Likewise, the BBC automaton is in the RFT class in $d = 1$, but its phase transition is discontinuous for $d > 1$. There is no way to predict this behavior from its field-theoretical representation. We must continue in our effort to find simple rules for the classification of nonequilibrium phase transitions.

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